TRANSFORMERS

Transformer is an ac machine; the main advantage of alternating currents over direct currents is that, the alternating currents can be easily transferable from low voltage to high or high voltage to low.

Alternating voltages can be raised or lowered as per the requirements in the different stages of electrical network as generation, transmission, distribution and utilization. This is possible with a device called transformer.

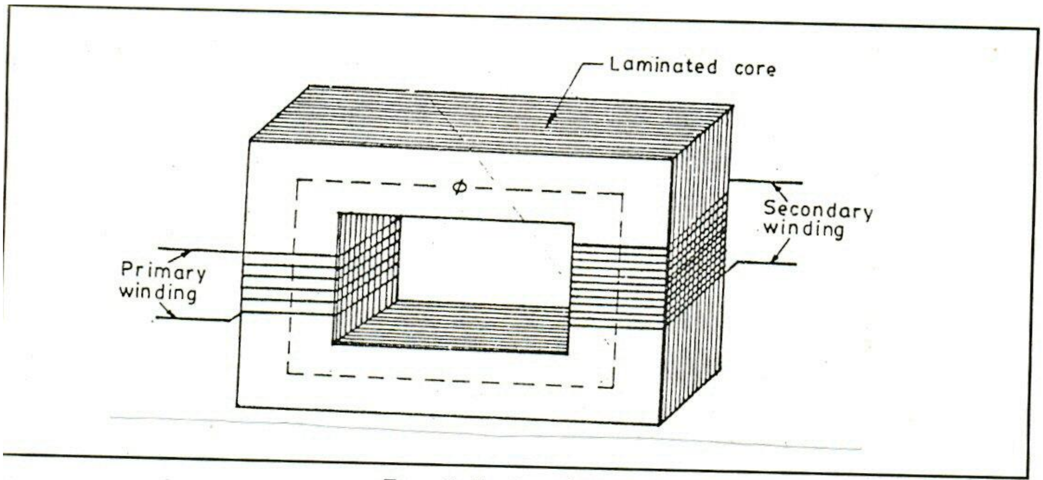
Definition:

Transformer is a static piece of apparatus by means of which electrical power is transferred from one alternating current circuit to another with the desired change in voltage or current and without any change in the frequency.

Principle of operation:

It is a static machine and it works on the principle of statically induced emf. It consists of:

- Magnetic circuit and
- Electric circuit



Two winding transformer

Two separate electrical windings are linked through a common magnetic circuit. The two electrical windings are isolated from each other.

The coil in which electrical energy is fed is called primary winding while the other from which electrical energy is drawn out is called secondary winding.

The primary winding has N_1 number of turns while secondary winding has N_2 number of turns.

When primary winding is excited by alternating voltage say V_1, it circulates alternating current I_1 through it. This current produces an alternating flux 'ϕ' which completes its path through the common magnetic core.

This flux links with both the windings. Because of this, it produces self induced emf E_1 in the primary winding while due to mutual induction i.e. due to flux produced by primary linking with secondary, it produces induced emf E_2 in secondary winding.

These emf's are:

$$E_1 = -N_1 \frac{d\phi}{dt} \qquad E_2 = -N_2 \frac{d\phi}{dt}$$

If now secondary circuit is closed through the load, the mutually induced emf in the secondary winding circulates current through the load. Thus electrical energy is transferred from primary to secondary with the help of magnetic core. A voltage V_2 appears across the load. Hence V_1 is the supply voltage, while V_2 is the secondary voltage when load is connected, then:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad or \quad V_2 = V_1\left(\frac{N_2}{N_1}\right)$$

$$k = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \text{ Transformation ratio}$$

If k > 1, then $V_2 > V_1$, transformer is called step up transformer.

If k < 1, then $V_2 < V_1$, transformers is called step down transformers.

If k = 1, then $V_2 = V_1$, then transformer is called one to one transformer.

The current flowing through primary is I_1 and when load is connected current I_2 flows through secondary voltage. The power transfer from primary to secondary remains the same. Assuming both primary and secondary power factor to be the same, we can write:

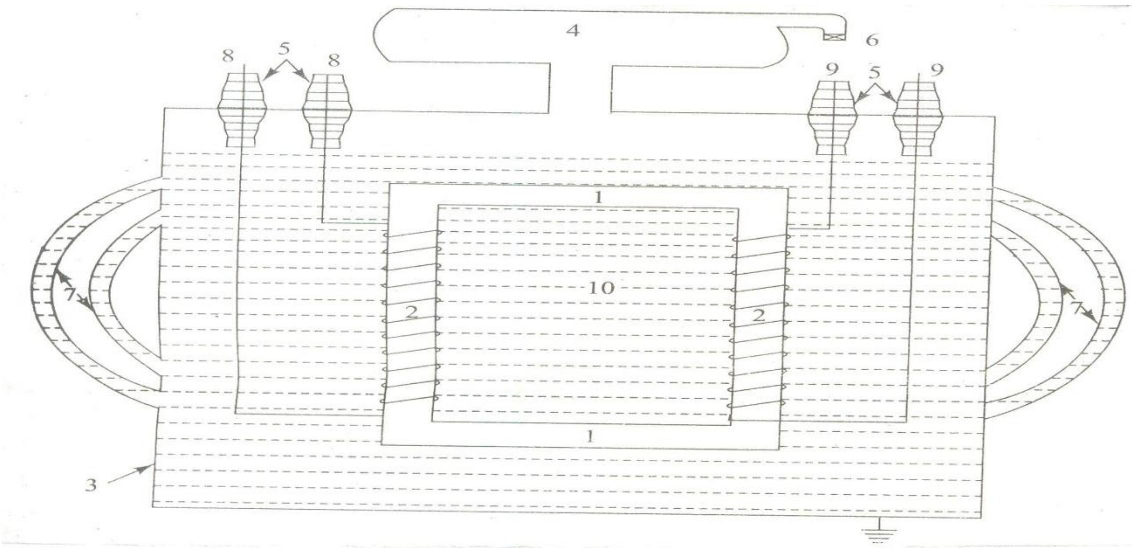
Power input to primary = Power output from secondary

$$V_1 I_1 = V_2 I_2 \qquad \boxed{\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{E_1}{E_2}}$$

Construction of a transformer:

There are basic parts of a transformer:

(1) Magnetic core
(2) Windings or coils
(3) Tank or Body
(4) Conservator Tank
(5) Breather
(6) Radiator
(7) Bushings



1. Laminated core
4. Conservator
7. Radiators
10. Transformer oil
2. Windings
5. Bushing
8. Winding leads (input)
3. Tank
6. Breather
9. Winding leads (output)

Core: The transformer core is made of silicon steel or sheet steel with 4 % silicon. The sheets are laminated and are coated with Oxide layer to reduce iron losses. The thickness of lamination is 0.35mm for 60 Hz and 5mm for 25Hz.

The core of the transformer is either square or rectangular in size. It is further divided in to two parts. The vertical portion on which coils are wound is called limb while the top and bottom horizontal portion is called yoke of the core.

The core is made up of laminations. Because of laminated type of construction, eddy current losses get minimized. Generally high-grade silicon steel laminations are used. These laminations are insulated from each other by using insulation like varnish.

The purpose of the core is to provide magnetic path of low reluctance between the two windings so that the total flux produced by one of the winding will be linked fully with the other winding without any leakage.

Windings: A transformer has two windings. The winding which receives electrical energy is called Primary winding and the winding which delivers electrical energy is called Secondary winding. Windings are generally made up of High grade copper. The windings are provided with insulation so that one winding may not come in contact with the other winding. Generally cotton, Paper and Oxide layer is used as insulating medium.

Tank or Body: It is part which is meant to carry the transformer and the oil used in the transformer. The tank used for a transformer should be air tight so that moisture should also not enter into the tank so as to maintain the properties of the transformer oil.

Transformer Oil: It is the most important part of a transformer which decides the life of a transformer. The oil that is used in a transformer should be safe guarded properly so as to have a good life for a transformer.

Conservator Tank: When a transformer is oil filled and self cooled the oil in the tank is subjected to heat and thus will naturally expand and contract due to variations in the load current and is also subjected to seasonal variations. The conservator tank provides the means for the oil to settle down by expanding under heavy loads.

Breather: Transformer oil should not be exposed to atmosphere directly because it may absorb
Moisture and dust from the environment and may loose its electrical properties in a very short time. To avoid this from happening a breather is provided. The breather completely prevents the moisture and dust from coming into contact with the oil in the conservator tank when it expands or contracts.

Bushings: The purpose of Bushings is to provide proper insulation for the output leads to be taken out from the transformer tank. Bushings are generally of two types.
a)Porcelain type which are used for voltage ratings uptp33kv
b)Condensor type and Oil filled type are used for rating above 33kv

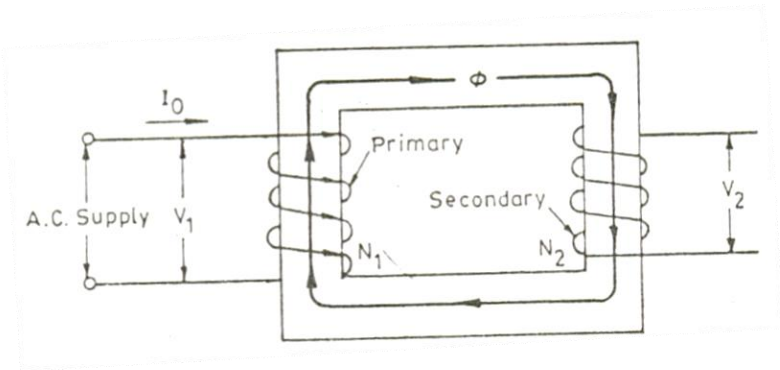
Radiator: These are meant to increase the surface area of the tank also to provide a path for the circulating of the transformer oil.

Types Of Transformers:

The transformers are classified based on the relative position or arrangement of the core & the windings, based on cooling and based on Voltage.

a)Based on arrangement of the core & the windings transformers are classified as

- Core type
- Shell type

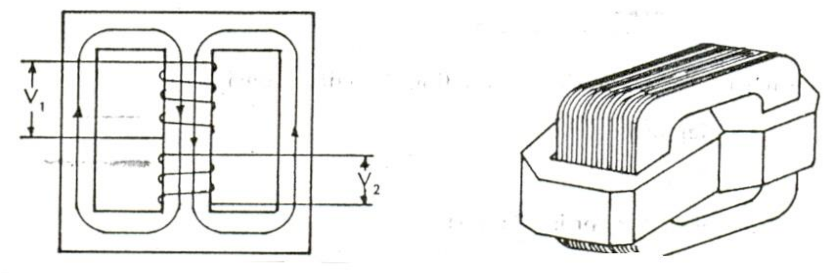


Core type Transformer

It has a single magnetic circuit. In this type, winding encircles the core, coils used are of cylindrical type. Such coils are wound in helical layers with different layers insulated from each other by paper, cloth, mica, etc. Core is made up of large number of thin laminations to reduce eddy current losses.

The windings are uniformly distributed over two limbs and hence natural cooling is more effective.

The coils can be easily removed by removing laminations of top yoke for maintenance.



Shell type transformer

It has a double magnetic circuit. In this type core encircles the most part of the winding. The core is again laminated one and while arranging the laminations, care is taken that all joints at alternate layers are staggered.

This is done to avoid narrow air gap at the joint, right through the cross section of the core.Such joints are called as overlapped. The coils are multi-layered disc type or sandwich type coils and are placed on only one limb and are surrounded by the core. So natural cooling does not exist.

b)Based on Voltage

- Step up
- Step Down

  Step up transformer is a transformer where the Output Voltage is greater than input Voltage i,e $V2>V1$
  Step down transformer is a transformer where the Output Voltage is less than input Voltage i,e $V2<V1$

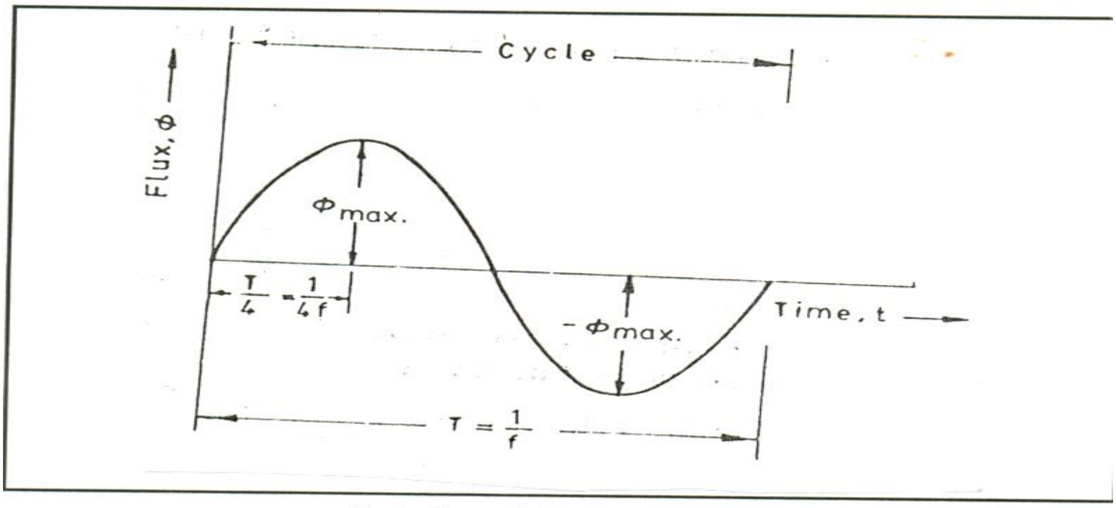
  Transformation Ratio $=V2/V1=N2/N1$
  Where N2,N1 are number of turns in secondary and primary windings.

c)Based on cooling:

- Oil cooled
- Oil filled water cooled
- Air cooled

EMF equation of a transformer:

Primary winding is excited by a voltage, which is alternating in nature. This circulates current through primary, which is also alternating and hence the flux produced is also sinusoidal in nature.



Variation of sinusoidal flux

Let ϕ = flux in the core

$\phi_m = B_m \times A$

N_1 = number of turns in the primary winding

N_2 = number of turns in the secondary winding

f = frequency of ac input in Hz.

The flux increases from its zero value to maximum value ϕ_m in one quarter of the cycle i.e., in ¼ f second.

Average rate of change of flux = $\frac{\phi_m}{¼ f}$ = 4 f ϕ_m wb/sec

Rate of change of flux per turn means induced emf in volts

Average emf / turn = 4 f ϕ_m volt.

If flux ϕ varies sinusoidally then rms value of induced emf is obtained by multiplying the average value with the form factor.

$$\text{Form factor} = \frac{rms\ vaue}{average\ value} = 1.11$$

rms value of emf/ turn = 1.11 x 4 f ϕ_m = 4.44 f ϕ_m

Now rms value of induced emf
in the whole of primary winding = induced emf / turn x number of primary turns

$$E_1 = 4.44\ f\ N_1\ \phi_m = 4.44\ f\ N_1\ B_m\ A$$

Similarly, rms value of emf induced in secondary is:

$$E_2 = 4.44\ f\ N_2\ \phi_m = 4.44\ f\ N_2\ B_m\ A$$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44 f\ \phi_m$$

That is, emf/ turn is same in both primary and secondary windings.

In an ideal transformers on no load $V_1 = E_1$ & $E_2 = V_2$

Where, V_2 is the terminal voltage.

Ideal transformers

Transformer is called ideal if it satisfies the following properties:

1. It has no losses
2. Its windings have zero resistance
3. Leakage flux is zero i.e. 100% flux produced by primary links with the secondary
4. Permeability of core is so high that negligible current is required to establish the flux in it.

An ideal transformer is one which has no loses i.e., its windings have no ohmic resistance, there is no magnetic leakage and hence which has no I^2R and core losses. In other words, **an ideal transformer consists of two purely inductive coils wound on a loss free core.**

Problems:

1. The maximum flux density in the core of a 250/3000 volts, 50 Hz single phase transformer is 1.2 wb/m^2. If the emf per turn in 8 volt, determine a) primary and secondary turns B) area of the core

   Solution: a) $E_1 = N_1$ x emf induced / turn

   $N_1 = 250/8 = 32$ $N_2 = 3000/8 = 375$

   b) $E_2 = 4.44\ f\ N_2\ B_m\ A$

   $3000 = 4.44 \times 50 \times 375 \times 1.2 \times A$

   $A = 0.03\ m^2$.

2. A single phase transformer has 400 primary and 1000 secondary turns. The net cross sectional area of the core is 60 cm^2. If the primary winding be connected to a 50 Hz supply at 520 V, calculate:

   a) peak value of flux density in the core
   b) the voltage induced in the secondary winding.

   Solution: $K = N_2/N_1 = 1000/400 = 2.5$

   a) $E_2/E_1 = k$ $E_2 = k\ E_1 = 2.5 \times 520 = 1300$ V

   b) $E_1 = 4.44\ f\ N_1\ B_m\ A$ $520 = 4.44 \times 50 \times 400 \times B_m \times 60 \times 10^{-4}$.

   $B_m = 0.976\ wb/m^2$.

3. A 25 KVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3000V, 50 Hz supply. Find the full load primary and secondary currents, the secondary emf and the maximum flux in the core. (Neglect leakage drops and no load primary

current).

Solution: $K = \frac{N_2}{N_1} = \frac{50}{500} = \frac{1}{10}$

Full load $I_1 = \frac{25000}{3000} = 8.33\,A$

Full load $I_2 = I_1/K = 10 \times 8.33 = 83.3$ A

emf per turn on primary side = 3000/500 = 6V

Secondary emf = 6 x 50 = 300 v
(or $E_2 = KE_1 = 3000 \times 1/10 = 300$ v)

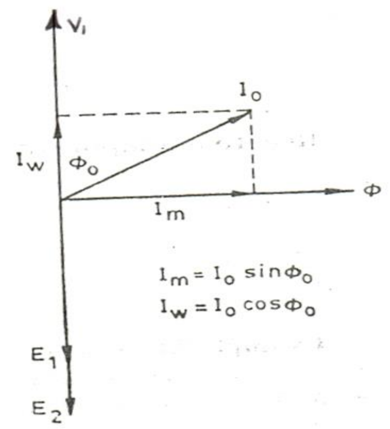
$E_1 = 4.44\, fN_1\, \phi_m$

$3000 = 4.44 \times 50 \times 500 \times \phi_m$

$\phi_m = 27$ mwb

Transformer on No-load:

An ideal transformer is one in which there were no core losses and copper losses. But practical conditions require that certain modifications be made in the foregoing theory. When an actual transformer is put on load, there is iron loss in the core and copper loss in the winding (both primary and secondary) and these losses are not entirely negligible.



No load vector diagram

Even when the transformer is on no-load the primary input current in not wholly reactive.

The primary input current under no load conditions has to supply (1) iron losses in the core i.e. hysteresis loss and eddy current loss and (2) a very small amount of copper loss in primary (there being no copper loss in secondary as it is open).

Hence, the no load primary input current I_o is not at 90° behind V_1 but lags it by an angle $\phi_0 < 90°$. No load input power $W_o = V_1\, I_o \cos\phi_o$, Where $\cos\phi_o$ is primary power factor under no load condition.

The primary current I_o has two components:

(1) One in phase with V_1. This is known as active or working or iron loss component I_o because it mainly supplies the iron loss plus a small quantity of primary copper loss.

$I_w = I_o \cos \phi_o$.

(2) The other component is in quadrature with V_1 and is known as magnetizing component I_μ because its function is to sustain the alternating flux in the core. It is watt-less.

$I\mu = I_o \sin \phi_o$

Obviously, I_o is the vector sum of I_w and I_μ, hence $I_o = \sqrt{I_\mu^2 + I_w^2}$

The following points should be noted carefully.

1. The no-load primary current I_o is very small as compared to the full-load primary current. It is about 1 percent of the full load current.

2. Owing to the fact that the permeability of the core varies with the instantaneous value of the exciting or magnetizing current is not truly sinusiodal. As such, it should not be represented by a vector because only sinusoidally varying quantities are represented by rotating vectors.

3. As I_o is very small, the no load primary copper loss is negligibly small which means that **no load primary input is practically equal to the iron loss in the transformer.**

4. As it is principally, the core loss which is responsible for shift in the current vector, angle **ϕ_o is known as hysteresis angle** of advance.

Problem:

a) A 2,200/200–V transformer draws a no load primary current of 0.6 A and absorbs 400 watts. Find the magnetizing and iron loss currents.

b) A 2200 / 250-V transformer takes 0.5 A at a p.f. of 0.3 on open circuit. Find magnetizing and working component of no-load primary current.

a) Iron – loss current = $\dfrac{no-load\ \ input\ \ in\ watts}{primary\ \ voltage}$ $\dfrac{V_1\ I_o \cos\phi\ _0}{V_1}$

$I_w = 400/2200 = 0.182$ A

$Now\ \ I_0^2 = I_w^2 + I_\mu^2$

Magnetizing component $I\mu = \sqrt{(0.6)^2 - (0.182)^2} = 0.572\ A$

c) $I_o = 0.5$

$\cos\phi_0 = 0.3$

$I_w = I_0 \cos \phi_0 = 0.5 \times 0.3 = 0.15$ A

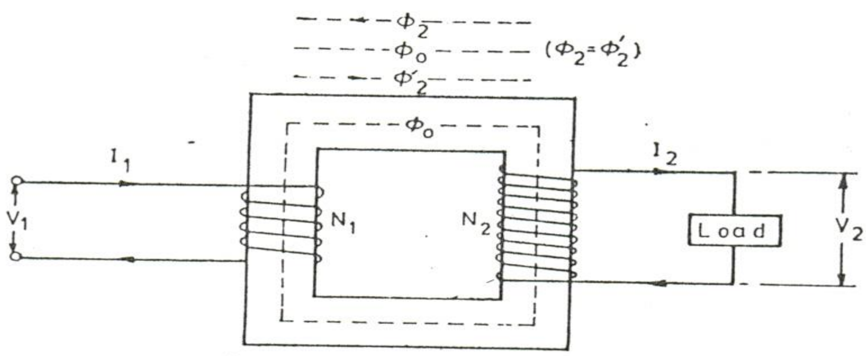
$I_\mu = \sqrt{(0.5)^2 - (0.15)^2} = 0.476\ \ A$

Transformer on load:

When the secondary is loaded, the secondary current I_2 is set up. The magnitude and phase of I_2 w.r.t. V_2 is determined by the characteristics of load. Current I_2 is in phase with V_2 if load is non-inductive, it lags if load is inductive and it leads if load is capacitive. The secondary current sets up its own mmf (N_2I_2) and hence its own flux ϕ_2 which is in opposition to the main flux ϕ which is due to I_0.

The secondary ampere turns N_2 I_2 are known as demagnetizing ampere turns. The opposing secondary flux ϕ_2 weakens the primary flux ϕ hence primary back emf E_1 tends to be reduced. For a moment V_1 gains the upper hand over E_1 and hence causes more current to flow in primary. Let the additional primary current be I_2^1. It is known as load component of primary current. This current is in antiphase with I_2.

The additional primary mmf N_1 I_2^1 sets up its own flux ϕ_2^l which is in opposition to ϕ_2 (but is in the same directions as ϕ) and is equal to it in magnitude. Hence, the two cancel each outer out. So, the magnetic effects of secondary current I_2 are immediately neutralized by the additional primary current I_2^l which is brought into existence exactly at the same instant as I_2. **Hence whatever the load conditions, the net flux passing through the core is approximately the same as at no load.**



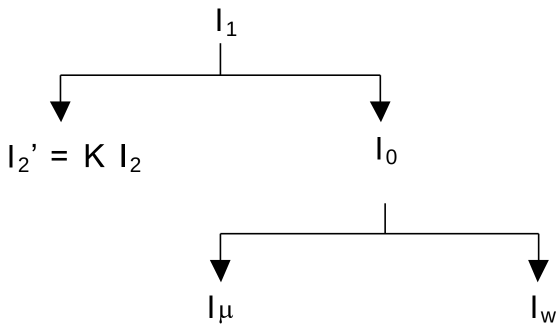
Transformer on load

$\phi_2 = \phi_2^l$

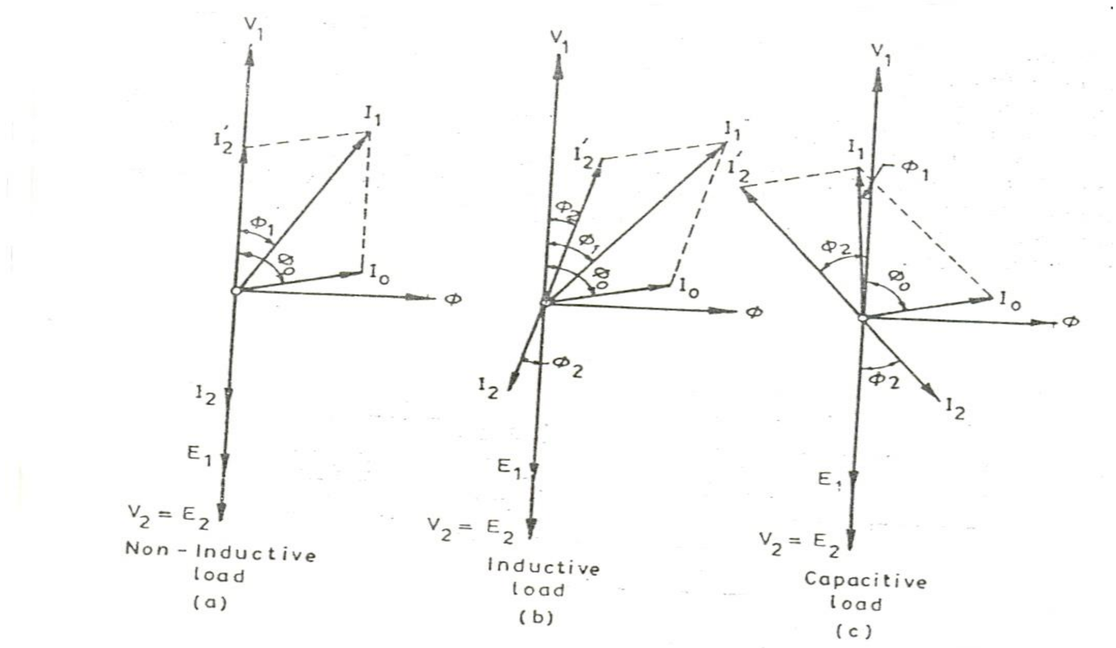
N_2 I_2 = N_1 I_2^l

$$I_2^l = \frac{N_2}{N_1} x I_2 = kI_2$$

Hence, when transformer is on load the primary winding has two currents in it; one is I_0 and at the other is I_2^1which is anti-phase with I_2 and k times in magnitude. The total primary current is the vector sum of I_0 and I_2^1.



The vector diagrams for a loaded transformer when load is non-inductive, inductive and when it is capacitive is shown. Voltage transformation ratio of unity is assumed so that primary vectors are equal to secondary vectors.



Vector Diagrams of a loaded transformer

With reference to the fig. (a) I_2 is secondary current in phase with E_2 (it should be v_2). It causes primary current I_2^l which is anti-phase with it and equal to it in magnitude (k = 1). The total primary current I_1 is the vector sum of I_0 and I_2^l and lags behind V_1 by an angle ϕ_1.

In (b) vectors are drawn for an inductive load. Here I_2 lags E_2 (actually V_2) by ϕ_2. Current I_2^1 is again in anti-phase with I_2 and equal to it in magnitude. I_1 is the vector sum of I_2^1 and I_0 and lags behind V_1 by ϕ_1. In (c) vectors are drawn for a capacitive load.

Problems:

A single phase transformer with a ratio of 440/110-V takes a no-load current of 5A at 0.2 p.f. lagging. If the secondary supplies a current of 120 A at a p.f. of 0.8 lagging, estimate the current taken by the primary.

$\cos \phi_2 = 0.8$

$\phi_2 = \cos^{-1} (0.8) = 36^0\ 54^I$

$\cos \phi_0 = 0.2$

$\phi_o = \cos (0.2) = 78^0\ 30^I$

Now $K = V_2 / V_1 = 110/440 = 1/4$

$I_2^I = K\ I_2 = 120 \times 1/4 = 30$ A

$I_0 = 5A$

Angle between $I_0 \times I_2^I$

$78^0\ 30^I - 36^0\ 54^I = 41^0\ 36^I$

Using parallelogram law of vectors, we get

$I_1 = \sqrt{5^2 + 30^2 + 2 x 5 x 30 x \cos 41^0\ 36^I} = 34.45$ A

Transformer with winding resistance but no magnetic leakage:

An ideal transformer posses no resistance, but in an actual transformer, there is always present some resistance of the primary and secondary windings. Due to this resistance, there is some voltage drop in the two windings. The result is that

1. The secondary terminal voltage V_2 is vectorially less than the secondary inducted emf E_2 by an amount $I_2\ R_2$ where R_2 is the resistance of the secondary winding. Hence, V_2 is equal to vector difference of E_2 and resistance voltage drop $I_2\ R_2$.

   $$\mathbf{V_2 = E_2 - I_2\ R_2.}$$

2. Similarly primary induced emf E_1 is equal to the vector difference of V_1 and $I_1\ R_1$ where R_1 is the resistance of the primary winding.

   $$\mathbf{E_1 = V_1 - I_1 R_1}$$

Transformer with resistance and leakage reactance:

The primary impedance: $Z_1 = \sqrt{R_1^2 + x_1^2}$

The secondary impedance: $Z_2 = \sqrt{R_2^2 + x_2^2}$

The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding.

In primary, the leakage reactance drop is $I_1\ X_1$, hence

$V_1 = E_1 + I_1 (R_1 + jX_1) = E_1 + I_1\ Z_1$. **$E_1 = V_1 - I_1\ Z_1$.**
Similarly $E_2 = V_1 + I_2 (R_2 + jX_2)$
$= V_2 + I_2\ Z_2$ **$V_2 = E_2 - I_2\ Z_2$.**

The vector diagrams for such a transformer for different kinds of loads are shown, in the diagrams, vectors for resistive drops are drawn parallel to current vectors where as reactive drops are drawn perpendicular to current vectors.

Vector Diagram

Steps to draw vector diagram:

1. Draw a horizontal line to represent flux line and a vertical line to represent voltage axis.

2. Draw a line OA in the III quadrant at some inclination to represent V_2 the terminal voltage.

3. Let I_2 be the load current in line with V_2 (resistive load)

4. At the tip of V_2, draw AB parallel to I_2 to represent I_2 R_2 drop (in line with V_2)

5. At 'B' draw BC at right angles to AB to represent I_2 X_2 meeting the vertical line at 'C' then OC is equal to secondary induced emf E_2.

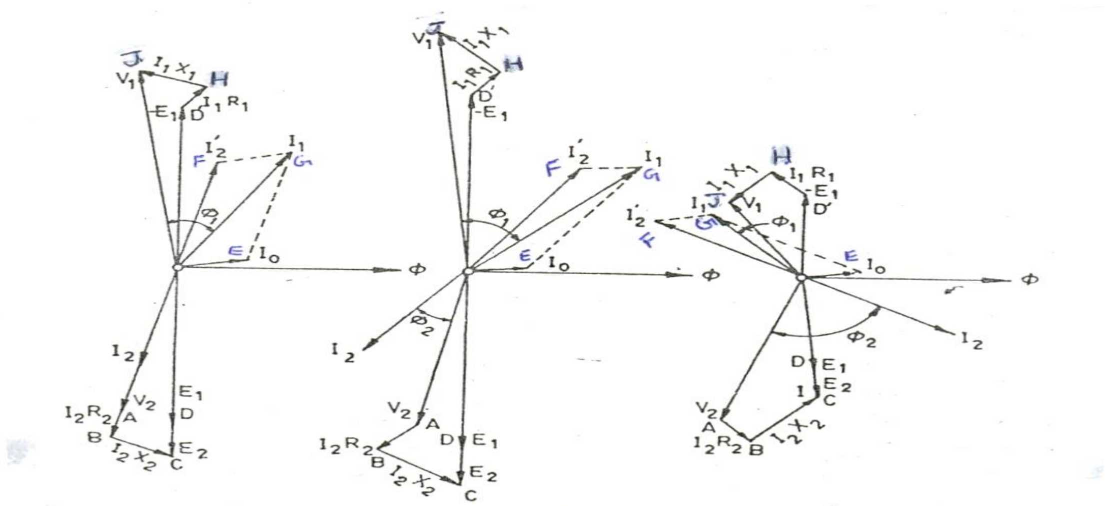
6. Produce 'O' backwards to 'D' such that OD = OC/k then OD represents the primary induced emf.

7. Produce the current line I_2 backwards to F such that OF = KI_2. Let 'OE' be equal to the load current 'I_0' at angle of 'ϕ_0' from E_1.

8. Construct the parallelogram OEGF then OG represents the primary current I_1.

9. At the tip of E_1 i.e. at D, draw DH equal to I_1R_1, parallel to OG. At 'H' draw 'HJ' perpendicular to DH to I_1X_1 drop

10. V_1 is obtained by adding vectorially the impedance drop I_2Z_2 to $-E_1$.



Vector Diagrams of a Loaded Transformer

Equivalent Resistance:

A transformer with primary and secondary winding resistance of R_1 and R_2 can be transferred to any one of the two windings. The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only.

A resistance of R_2 in secondary is equivalent to R_2/k^2 in primary. The value R_2/K_2 will be denoted by R_2^l – the equivalent secondary resistance as referred to primary.

The copper loss in the secondary is $I_2^2 R_2$. This loss is supplied by the primary which takes a current of I_1. Hence, if R_2^l is the equivalent resistance in primary which would have caused the same loss as R_2 in secondary, then

$$I_1^2 R_2^l = I_2^2 R_2$$

$$R_2^l = (I_2/I_1)^2 R_2.$$

If no load current I_0 is neglected, then $I_2/I_1 = 1/k$

Hence, $R_2^l = R_2/k^2$

Similarly equivalent primary resistance as refereed to secondary is $R^1{}_1 = k^2 R_1$.

The resistance $R_1 + R_2^l = R_1 + R_2/K^2$ is known as the equivalent or effective resistance of the transformer as referred to primary and is designated as

$$\mathbf{R_{01} = R_1 + R_2^1 = R_1 + R_2/k^2.}$$

Similarly the equivalent resistance of the transformer as refereed to secondary

$$\mathbf{R_{02} = R_2 + R_1^1 = R_2 + k^2 R_1.}$$

1. When shifting resistance to secondary multiply it by k^2.
2. When shifting resistance to primary, divide by k^2.

Leakage Reactance:

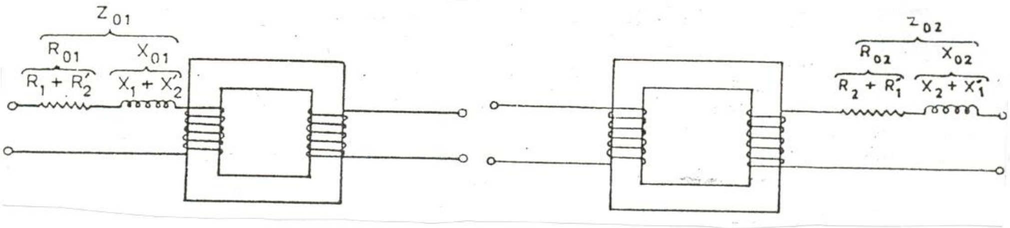
Leakage Reactance can also be transferred from one winding to the other in the same way as resistance.

$$X_2' = \frac{X_2}{k^2} \quad and \quad X_1' = k^2 X_1 \qquad X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{k^2}$$

$$X_{02} = X_2 + X_1' = X_2 + k^2 X_1$$

Total impedance

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} \qquad Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$



Total impedance transferred to primary and secondary side

Example: A 30 KVA, 2400/120 V, 50 Hz transformer has a high voltage winding resistance of 0.1 Ω and leakage reactance of 0.22 Ω the low voltage winding resistance is 0.035 Ω and the leakage reactance is 0.012 Ω, find the equivalent winding resistance, reactance and impedance as referred to

(a) high voltage side (b) low-voltage side

K = 120/2400 = 1/20
R_1 = 0.1 Ω X_1 = 0.22 Ω R_2 = 0.035 Ω and X_2 = 0.012 Ω

a) high voltage side is the primary side

Hence values as refereed to primary side are

$$R_{o1} = R_1 + R_2^1 = R_1 + \frac{R_2}{k^2} = 0.1 + \frac{0.035}{(1/20)^2} = 14.1\ \Omega$$

$$X_{o1} = X_1 + X_2^1 = X_1 + \frac{X_2}{k^2} = 0.22 + \frac{0.12}{(1/20)^2} = 5.02\ \Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} + \sqrt{14.1^2 + 5.02^2} = 15\Omega$$

b) $$R_{o2} = R_2 + R_1^1 = R_2 + k^2\ R_1 = 0.035 + (1/20)^2 x\ 0.1 = 0.03525\ \Omega$$

$$X_{o2} = X_2 + X_1^1 = X_2 + k^2\ X_1 = 0.012 + (1/20)^2 x\ 0.22 = 0.01255\ \Omega$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.325^2 + 0.01255\ ^2} = 0.0374\ \Omega$$

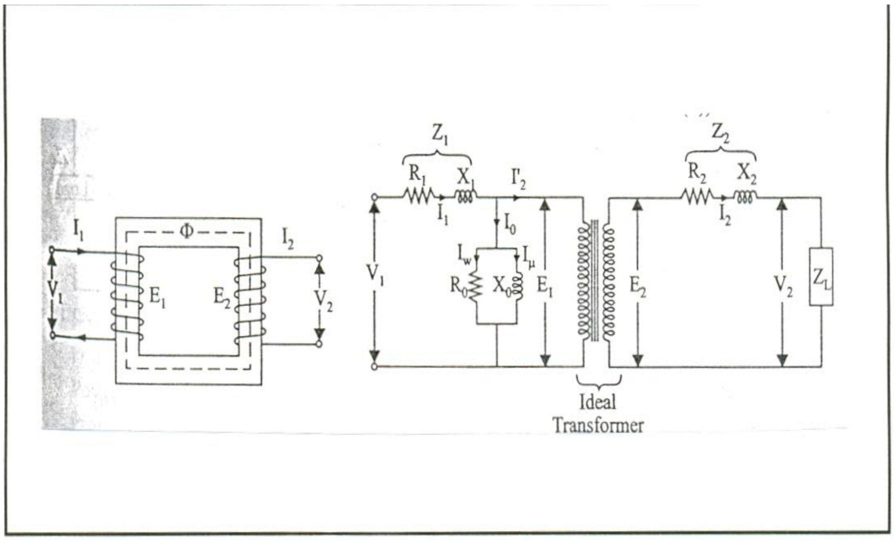
$$Z_{02} = k^2\ Z_{01} = (1/20)^2\ x\ 15 = 0.0375\ \Omega$$

Equivalent circuit:

The transformer shown in (a) is resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding whose only function is to transform the voltage. The no-load current I_0 consists of two components, I_w and I_μ therefore, I_0 is splitted into two parallel branches. The current I_μ accounts for the core-loss and hence is shown to flow through resistance R_0.The current I_μ represents magnetizing component and is shown to flow through a pure reactance X_0 .The value of E_1 is obtained by subtracting vectorially I_1Z_1 from V_1.

$X_o = E_1/I_\mu$ and $R_o = E_1/I_w$

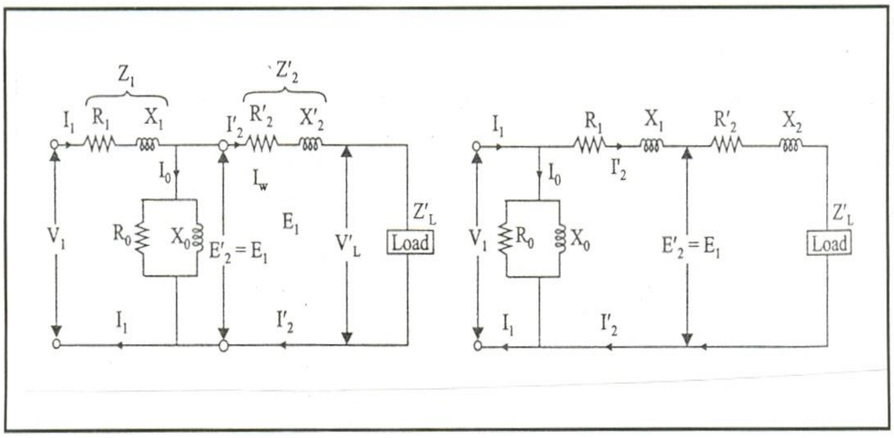
E_1 & E_2 are related to each other by the expression $E_2 / E_1 = N_2/N_1 = k$



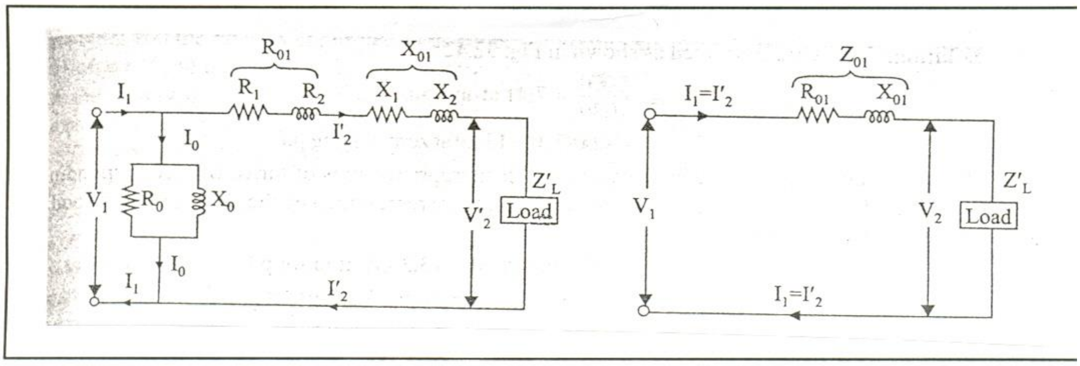
To make transformer calculations simple, it is preferable to transfer voltage, current and impedance either to the primary or to the secondary. The primary equivalent of the secondary induced voltage is $E_2^I = E_2/K = E_1$. Similarly, primary equivalent of secondary terminal and output voltage is $V_2^I = V_2/K$. Primary equivalent of the secondary current $I_2^I = KI_2$. For transferring secondary impedance to primary, K^2 is used

$R_2^{'} = R_2/K^2$ $X_2^{'} = X_2/K^2$ $Z_2' = Z_2/K^2$

A simplified equivalent circuit of a transformer, as referred to primary side is shown. A simplification can be made by transferring the exciting circuit across the terminal as below:



Further simplification may be achieved by omitting I_0 altogether as shown below:



Transformer tests:

The performance of a transformer can be calculated on the basis of its equivale circuit which contains four main parameters, the equivalent resistance R_{01} a referred to primary, the equivalent leakage reactance X_{01} as referred to primar the core loss conductance G_0 (resistance R_0) and the magnetizing susceptance I (reactance X_0)

These constants or parameters can be easily determined by two tests

(1) Open-circuit test

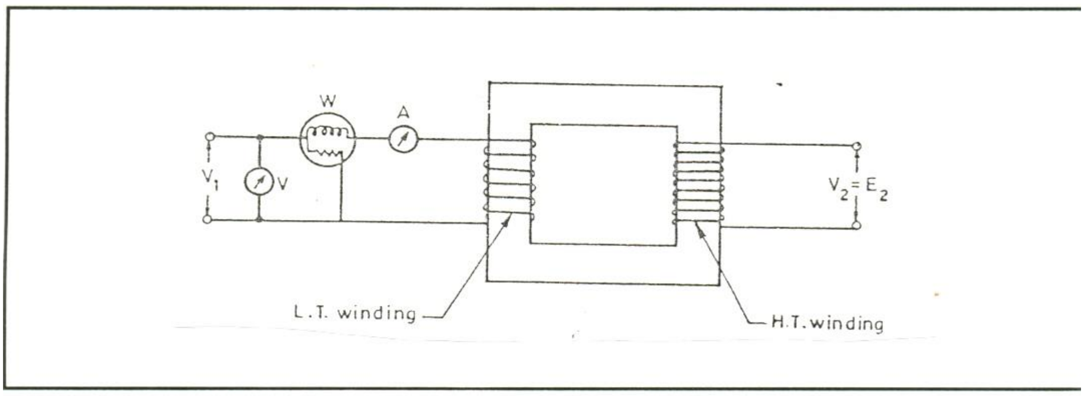
(2) Short-circuit test.

Open circuit or No-load test

The purpose of this test is to determine no-load loss or core loss and no load which is helpful in finding X_0 and R_0. One winding of the transformer i.e. **a hig voltage side is left open** and the other is connected to its supply of norm voltage and frequency.

A wattmeter W, voltmeter V and an ammeter A are connected in the low-voltag winding, i.e., primary winding. With normal voltage applied to primary, norm flux will be set up in the core, hence normal iron losses will occur which ar recorded by the wattmeter.

As the primary no-load current I_0 is small, copper loss is negligibly small primary and nil in secondary. Hence, the wattmeter reading represents practical the core loss under no-load condition.



$W = V_1 I_o \cos \phi_o$.

$\cos \phi_o = W/V_1 I_o$

$I_\mu = I_o \sin \phi_o$

$I_w = I_o \cos \phi_o$

$X_o = V_1/I_\mu$ & $R_o = V_1/I_w$.

$I_o = V_1 Y_o$

$Y_o = I_o/V_1$

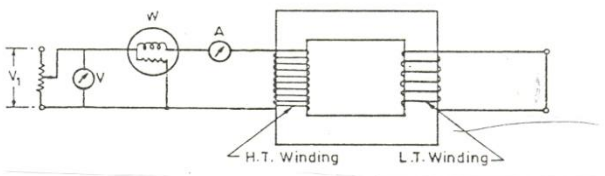
G_o is given equation $W = V_1^2 G_o$

$G_o = W/V_1^2$

$B_o = \sqrt{Y_0^2 - G_0^2}$

Short-circuit or Impedance Test:

In this test, one winding usually the **low-voltage winding is solidly short – circuited by a thick conductor.**



This method is used to find the following parameters.

1. Equivalent impedance (Z_{o1} or Z_{o2}), leakage reactance (X_{o1} or X_{o2}) and total resistance (R_{o1} or R_{o2}) of the transformer as refereed to the winding in which the measuring instruments are placed.

2. Cu loss at full-load (at any desired load). This loss is used in calculating the efficiency of the transformer.

3. Knowing Z_{01} or Z_{02}, the total voltage drop in the transformer as referred to primary or secondary can be calculated and hence regulation of the transformer determined.

A low voltage at correct frequency is applied to the primary and is cautiously increased till full-load currents are flowing both in primary and secondary. Since, in this test the applied voltage is a small percentage of the normal voltage, the mutual flux ϕ produced is also a small percentage of its normal value.
Hence, core losses are very small with the result that the wattmeter reading represents the full load copper loss or $I^2 R$ loss for the whole transformer i.e. both primary copper loss and secondary copper loss.

If V_{sc} is the voltage required to circulate rated load currents, then $Z_{01} = V_{sc} / I_1$

Also, $W = I_1^2 R_{01}$ $R_{01} = W/I_1^2$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

Losses in the transformer:

Losses that occur in a transformer are:

- Core or iron losses
- Copper losses

Core losses:

It includes both the hysteresis loss and eddy current loss. These losses are minimized by using steel of high silicon content for the core and by using very thin laminations.

Iron or core loss is found from the O.C test. The input of the transformer when on no-load measures the core loss.

Hysteresis loss:

When a magnetic material is subjected to repeated cycles of magnetization and demagnetization it results into disturbance and there will be loss of energy and this loss of energy appears as heat in the magnetic material. This is called as hysteresis loss.

Hysteresis loss = $k_h B_m^{1.6}$ f x volume, watts

Where, K_h = constant
B_m = maximum flux density
f = frequency

The hysteresis loss can be reduced by using thin laminations for the core.

Eddy current loss:

Due to alternating fluxes linking with the core, eddy currents get induced in the laminations of the core. Such eddy currents cause the eddy current loss in the core and heat up the core.

Eddy current loss can be reduced by selecting high resistivity material like silicon. The most commonly used method to reduce this loss is to use laminated construction to construct the core. Core is constructed by stacking thin pieces known as laminations. The laminations are insulated from each other by thin layers of insulating material like varnish, paper, mica. This restricts the paths of eddy currents, to respective laminations only. So area through which currents flow decreases, increasing the resistance and magnitude of currents gets reduced.

Eddy current loss = $K_e B_m^2 f^2 t^2$, watts

Where, K_e = constant
B_m = maximum flux density

f = frequency
t = thickness of the laminations

Copper loss:

This loss is due to the ohmic resistance of the transformer windings.

Total copper loss = $\mathbf{I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02}}$

Copper loss is proportional to $(\text{current})^2$ or $(\text{KVA})^2$

Copper loss at half the full-load is one fourth of that at full-load.

Efficiency:

$$\frac{output}{input} = \frac{output}{output + cu.loss + ironloss}$$

$$\frac{input - losses}{input} = 1 - \frac{losses}{input}$$

Copper loss = $I_1^2 R_{01}$ or $I_2^2 R_{02} = W_{cu}$

Iron losses = hysteresis loss + eddy current loss = $W_h + W_e = W_i$

primary input = $V_1 I_1 \cos \phi_1$

$$\eta = \frac{V_1 I_1 \cos\phi_1 - losses}{V_1 I_1 \cos\phi_1} = \frac{V_1 I_1 \cos\phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos\phi_1}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos\phi_1} - \frac{W_i}{V_1 I_1 \cos\phi_1}$$

Differentiating both sides with respect to I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos\phi_1} + \frac{W_i}{V_1 I_1^2 \cos\phi_1}$$

For η to be maximum, $\frac{d\eta}{dI_1} = 0$. Hence

$$\frac{R_{01}}{V_1 \cos\phi_1} - \frac{W_i}{V_1 I_1^2 \cos\phi_1} \quad \text{or} \quad W_i = I_1^2 R_{01} \text{ or } I_2^2 R_{02}$$

Cu loss = Iron loss

The output current corresponding to maximum efficiency is $I_2 = \sqrt{\left(\frac{W_i}{R_{02}}\right)}$

It is this value of output current, which will make the Cu-loss equal to the iron loss.

1. If we are given iron loss and full load Cu loss, then the load at which two losses would be equal

$$\text{Full load} \times \sqrt{\frac{\text{Iron loss}}{\text{F.L. cu loss}}}$$

2. Efficiency at any load

$$\eta = \frac{x * full-load\ kVA * p.f}{(x * full-load\ kVA * p.f.) + W_{cu.} + W_i} X\ 100$$

Transformer rating in KVA:

Cu loss of a transformer depends on current and iron loss on voltage. Hence, total transformer loss depends on volt-ampere (VA) and not on phase angle between voltage and current i.e., it is independent of load power factor. That is why rating of transformer is in KVA and not in KW.

Regulation of a Transformer

(1) When a transformer is loaded with a constant primary voltage, the secondary voltage decreases because of its internal resistance and leakage reactance.

Let, ${}_0V_2$ = secondary terminal voltage at no-load = E_2 = KE_1 = KV_1
because at no-load the impedance drop is negligible.

V_2 = secondary terminal voltage on full-load

The change in secondary terminal voltage from no - load to full-load is ${}_0V_2 - V_2$. This change divided by ${}_0V_2$ is known as regulation 'down'. If this change is divided by V_2 i.e., full-load secondary terminal voltage, then it is called regulation 'up'.

% regulation 'down' = $\dfrac{{}_0V_2 - V_2}{{}_0V_2} x\ 100$

% regulation 'up' = $\dfrac{{}_0V_2 - V_2}{V_2} x\ 100$

In further treatment, unless stated otherwise, regulation is to be taken as regulation 'down'.

The change in secondary terminal voltage from no-load to full-load, expressed as a percentage of no-load secondary voltage, is:

$$= V_r \cos\phi \pm V_x \sin\phi \text{ (approximately)}$$

where V_r= percent resistive drop = $100 \times \dfrac{I_2 R_{02}}{{}_0V_2}$

Vx = percent reactive drop = $100 \times \dfrac{I_2 X_{02}}{{}_0V_2}$

or more accurately = $(V_r \cos\phi \pm V_x \sin\phi) + 1/200\ (V_x \cos\phi \mp V_r \sin\phi)^2$

% regulation = $V_r \cos\phi \pm V_x \sin\phi$

(2) The regulation may also be explained in terms of primary values.

The secondary no-load terminal voltage as referred to primary is $E_2^{l} = E_2/K = E_1 = V_1$ and if the secondary full-load voltage as referred to primary is V_2^{l} ($= V_2/K$)

$$\% \text{ regulation} = \frac{V_1 - V_2^1}{V_1} x\ 100$$

If angle between V_1 and V_2^{l} is neglected, then the value of numerical difference $V_1 - V_2^{l}$ is given by ($I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi$) for lagging p.f.

$$\% \text{ regulation} = \frac{I_1\ R_{01}\ \cos\phi + I_1\ X_{01}\ \sin\phi}{V_1} x\ 100 = v_r\ \cos\phi\ + v_x\ \sin\phi$$

where,

$$\frac{I_1\ R_{01}\ x\ 100}{V_1} = v_r \quad and \quad \frac{I_1\ X_{01}\ X\ 100}{V_1} = v_x$$

if angle between V_1 and V_2^1 is not negligible, then

$$\% \text{ regulation} = (v_r\ \cos\phi \pm v_x\ \sin\phi) + \frac{1}{200}(v_x\ \cos\phi \mp v_r\ \sin\phi)^2$$

Problem 1:

Obtain the equivalent circuit of a 200/400-V,50Hz, 1-phase transformer from the following test data.

O.C. test: 200V, 0.7A, 70 W on low voltage side
S.C. test: 15V, 10A, 85 W on high voltage side

Calculate the secondary voltage when delivering 5KW at 0.5 p.f. lagging, the primary voltage being 200V.

Solution:

$V_1 I_0 \cos\varphi_0 = W_0$

200 x 0.7 x $\cos\varphi_0$ = 70w

$\cos\varphi_0 = 0.5$ $\sin\varphi_0 = 0.866$

$I_w = I_0 \cos\varphi_0$ =0.7x 0.5 = 0.35A

$I\mu = I_0 \sin\varphi_0$ =0.7x 0.866 = 0.606A

$R_0 = V_1/ I_w$ =200/0.35 = 571.4 Ω

$X_0 = V_1/ I\mu$ =200/0.606=330 Ω

From S.C. Test:

$Z_{02}=V_{sc}/I_2$ = 15/10 = 1.5 Ω

$Z_{01}=Z_{02}/K^2$ = 1.5/4 = 0.375 Ω

$I_2^2R_{02}$ = W; R_{02} = 85/100 = 0.85 Ω

$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$ = 0.31 Ω

$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2}$ = 1.24 Ω

Total transformer drop as referred to secondary = $I_2 (R_{02}\cos\varphi_2 + X_{02} \sin\varphi_2)$

= 15.6 (0.85 x 0.8 + 1.24 x 0.6) = 22.2V

∴ V_2 = 400 – 22.2 = **377.8 V**

Problem 2:

A 100 KVA, transformer has an iron loss of 1KW and a Cu loss on normal output current of 1.5 KW. Calculate the KVA loading at which the efficiency is maximum and its efficiency at this loading (a) at unity p.f (b) at 0.8 p.f. lagging

Solution:

Load KVA corresponding to maximum efficiency = full-load KVA X $\sqrt{\frac{iron\ loss}{F.L.Cu\ loss}}$

$= 100 \text{ X } \sqrt{\frac{1}{1.5}}$

= 82.3 kVA

(a) Total loss = 2 KW

$$\eta = \frac{x * full-load\ kVA * p.f}{(x * full-load\ kVA * p.f.) + W_{cu.} + W_i} \text{ X } 100 \ ;$$

where x = ratio of actual to full load KVA ;   ∴ x = 1;

$$= \frac{82.3 * 1}{(82.3 * 1) + 2} = 97.63\ \%$$

(b) At 0.8 p.f lagging

$$\eta = \frac{82.3 * 0.8}{(82.3 * 0.8) + 2} = 97.05\ \%$$